Humidity contribution to the refractive index structure function C_n^2.

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ABSTRACT

Humidity and C_n^2 data collected from the Chesapeake Bay area during the 2003/2004 period have been analyzed. We demonstrate that there is an unequivocal correlation between the data during the same time periods, in the absence of solar insolation. This correlation manifests itself as an inverse relationship. We suggest that C_n^2 in the infrared region is also function of humidity, in addition to temperature and pressure.

Keywords: Strength of turbulence, humidity, scintillation

1. INTRODUCTION

It has been known for some time[1] that the scintillation behaviour of point sources is a measure of the optical seeing in the atmosphere. What has been less well understood is the contribution of different environmental variables to optical seeing. Over the past decade, a great deal of study has been dedicated to clarifying this issue.

Comprehensive treatments of the theory of wave propagation in random media are given in Tatarskii's seminal works.[2,3] More recent developments are described in Tatarskii et al..[4] Some of the simplest models based on these complex works are well known and available in the literature: Greenwood,[5] Hufnagel–Valley,[6] SLC-Day and SLC-Night.[7] These models are used to predict the strength of weak clear air turbulence's refractive index structure function, C_n^2, but in all cases they have major failings: either they are too general and do not take into account local geography and environment (as in the former two) or they are too specific to a site (as in the latter two, which reference the median values above Mt. Haleakala in Maui, Hawaii).

A more recent numerical model known as PAMELA does attempt to account for geographical position and ambient climate factors. However, its inverse power windspeed term fails to explain some of the characteristics of C_n^2 during low wind conditions. It does an adequate job for characterizing horizontal and diagonal beam propagation within the atmospheric boundary layer.[8]

Despite the differences, the models agree in terms of the overall general behaviour of C_n^2. For example, it is to be expected that during the daylight hours, the C_n^2 trend will be dominated by the solar insolation and in those models that do account for day/night differences this is presented. The physical effect is evidenced in Oh,[9] where in many cases scintillometer measurements are seen to strongly follow the measured solar insolation function. When the sun sets however, it is less clear as to the predominant contributing factors. In an extension of earlier work, Oh presented indications of a possible anticorrelation effect between the ambient relative humidity and the value of C_n^2.

In this paper, we report on further analysis of the datasets obtained during that study to show that there is an unequivocal correlation in the absence of solar insolation in a littoral space.

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2. INSTRUMENTS AND ALGORITHMS

The C_n^2 and associated weather variable data was collected over a number of days during 2003 and 2004 at the Chesapeake Bay Detachment (CBD) of the Naval Research Laboratory.

The C_n^2 data was obtained with a commercially available scintillometer (model LOA-004) from Optical Scientific Inc, which serves as both a scintillometer and as an optical anemometer for winds transverse to the beam paths. The local weather parameters were determined by a Davis Provantage Plus (DP+) weather station. The LOA-004 had a sample rate of 10 seconds, while the DP+ was set at 5 minutes.

The LOA-004 instrument comprises of a single modulated infrared transmitter whose output is detected by two single pixel detectors. For these data, the separation between transmitter and receiver was 100-m. The path integrated C_n^2 measurements are determined by the LOA instrument by computation from the log–amplitude scintillation ($C_\chi(r)$) of the two receiving signals.[10, 11] The algorithm for relating $C_\chi(r)$ to C_n^2 is based on an equation for the log–amplitude covariance function in Kolmogorov turbulence by Clifford *et al.*,[12] which we repeat here

$$C_\chi(r) = 2.94 \int_0^1 du\sigma_T^2(u)[u(1-u)^{5/6}] \int_0^\infty dy y^{-11/6} sin^y \exp\{-\sigma_T^2[u(1-u)]^{5/6}F(y)\}J_0\left\{\left[\frac{4\pi yu}{(1-u)}\right]^{1/2} r\right\} \quad (1)$$

The terms in this equation are: r, the separation between two point detectors in Fresnel zones $\sqrt{\lambda L}$, with L being the path distance between source and detectors; y is the normalized spatial wavenumber; $u = z/L$ is the normalized path position; J_0 is the zero order Bessel function of the first kind and

$$\sigma_T^2(u) = 0.124k^{7/6}L^{11/6}C_n^2(u) \quad (2)$$
$$F(y) = 7.02y^{5/6}\int_{0.7y}^\infty d\xi\xi^{-8/3}[1-J_0(\xi)]$$

This can be better appreciated if we define a path weighting function $W(u)$ such that

$$C_\chi(r) = \int_0^1 duC_n^2(u)W(u) \quad (3)$$

for a point source and point receivers where

$$W(u) = 0.365k^{7/6}L^{11/6}[u(1-u)]^{5/6}\int_0^\infty dyg(u,y)J_0\left\{\left[\frac{4\pi yu}{(1-u)}\right]^{1/2} r\right\} \quad (4)$$

In the above expression, $g(u,y)$ carries the information related to C_n^2 for point source and point receivers. It can be modified to incorporate finite receiver and transmitter geometries.

Some comments are necessary at this point. The key assumptions made by the LOA-004 instrument in computing C_n^2 are:

- The turbulent power spectrum is Kolmogorov; the spatial power spectrum of temperature fluctuations $\Phi_T^2(k)$ and the humidity fluctuations $\Phi_H^2(k)$ are proportional to $k^{-5/3}$. This may not always be true, especially if the inner and outer scales are on the order of the relevant dimensions of the observing system.

- As a result of the previous assumption, the index of refraction structure function is assumed to be dependent only on the temperature structure function and pressure at optical frequencies.

What we demonstrate in this paper is that the LOA-004 measured C_n^2 function from the CBD experiment is indeed correlated with the humidity. The LOA-004's design is by no means optimal for extracting C_n^2, since its main purpose is to act as an anemometer. To that end we have an effort to evaluate contributions to the measurement error of C_n^2.
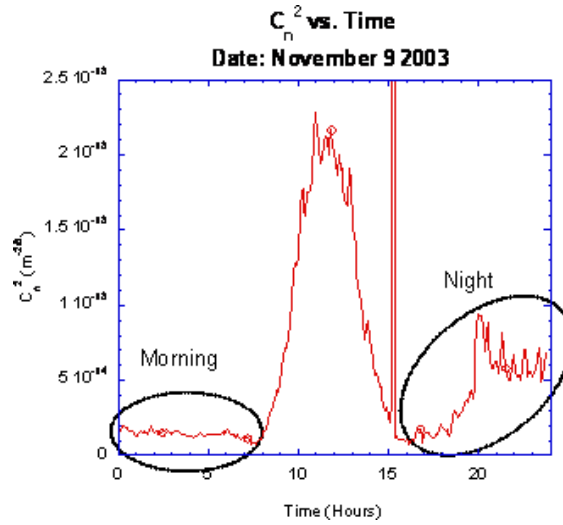
Figure 1. Morning and Night definitions for our purposes. See text for details.

3. ANALYSIS

The C_n^2 data was smoothed with a 60 point rolling average function. The effect of solar insolation was excluded from this study. We defined the morning and night portions of a 24 hour period as shown in Figure (1). Morning runs from midnight until sunrise (as corroborated by a solar irradiance measurement), while night runs from sunset to 23:59. As reported in Oh *et al.*[9] visual inspection of the valid time series data gives the impression that there is an approximate inverse relationship between C_n^2 and humidity. This can be appreciated in a more quantitative manner by graphing C_n^2 against humidity.

We chose data sets in which the temperature variations are no more than $\pm 15\%$ and the pressure change is at most 15 mbars over the time intervals of interest. The data sections were also selected to have no scattering effects due to snow or rain, and the wind was northerly (to within approximately $\pm 20°$, inflowing from the bay to land).

Given the aforementioned conditions, from the data available only a subset provided complete time series in both ambient weather variables and C_n^2. We were able to extract eight morning and evening runs, spanning seven days between November 2003 and March 2004 for the purpose of calculating the crosscorrelation, $\Gamma_{UV}(t + \delta t) = E[u(t + \delta t)v(t)]$, and cross covariance, $C_{UV}(t + \delta t) = E(u(t + \delta t) - \overline{u})(v(t) - \overline{v})$, between humidity and C_n^2 measureables. In these parameters, E represents expected value and $\overline{u}, \overline{v}$ are the mean values of the two random processes, considered stationary.

As can be seen from Figures (2 - 9), the C_n^2 against humidity correlograms all evidence a negative gradient. The tightness of the correlation is better examined in terms of the cross covariance. The results are normalized such that the value at zero time lag was unity for identically varying data.

3.1. Comments on Figures (2 - 9)

Fig.(2) The correlogram shows an approximately even dispersion along the length of the best fit trendline. The cross covariance lacks symmetry.

Fig.(3) The correlogram shows the tightest correlation between the data series of all the plots and the cross covariance is quite symmetric, although highly structured.

Fig.(4) The correlogram is fairly even, with some larger dispersion possibly occuring around 70% of humidity. The cross covariance is symmetric.
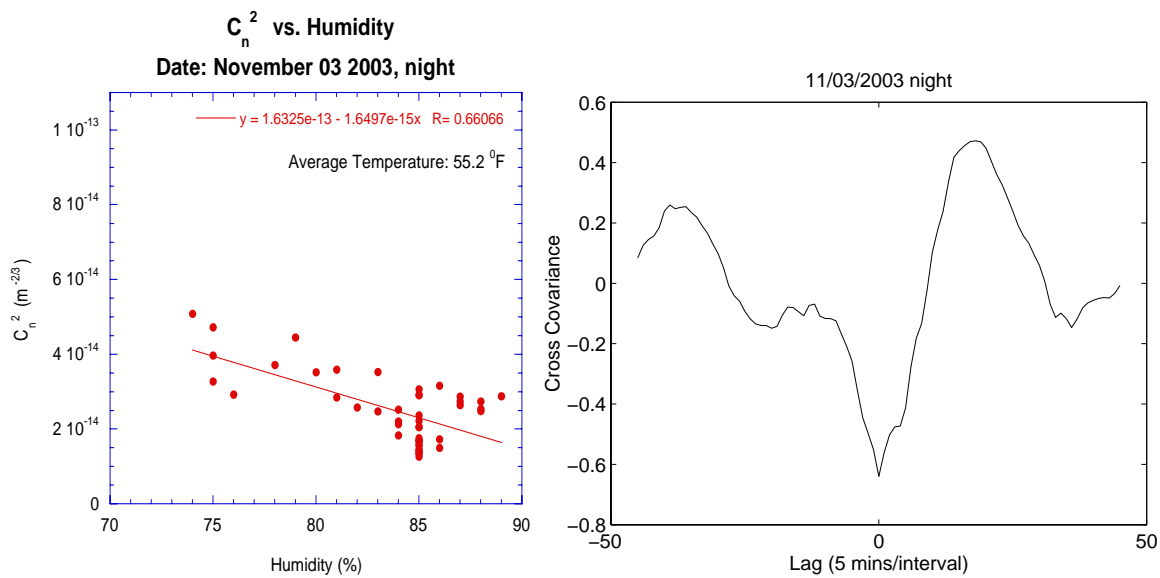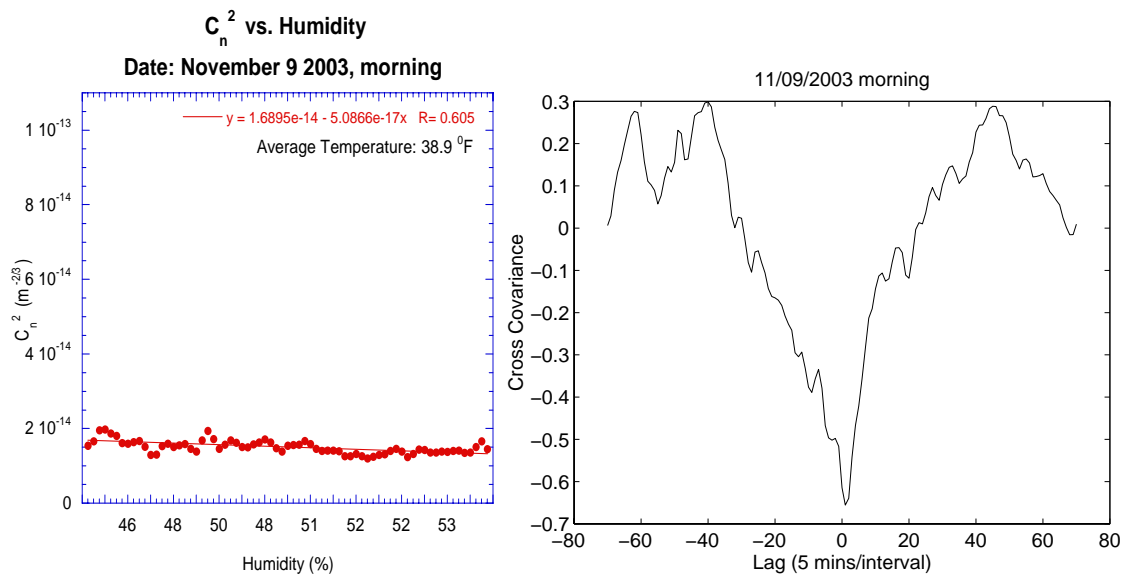
Figure 2. Nov 3 2003, Night
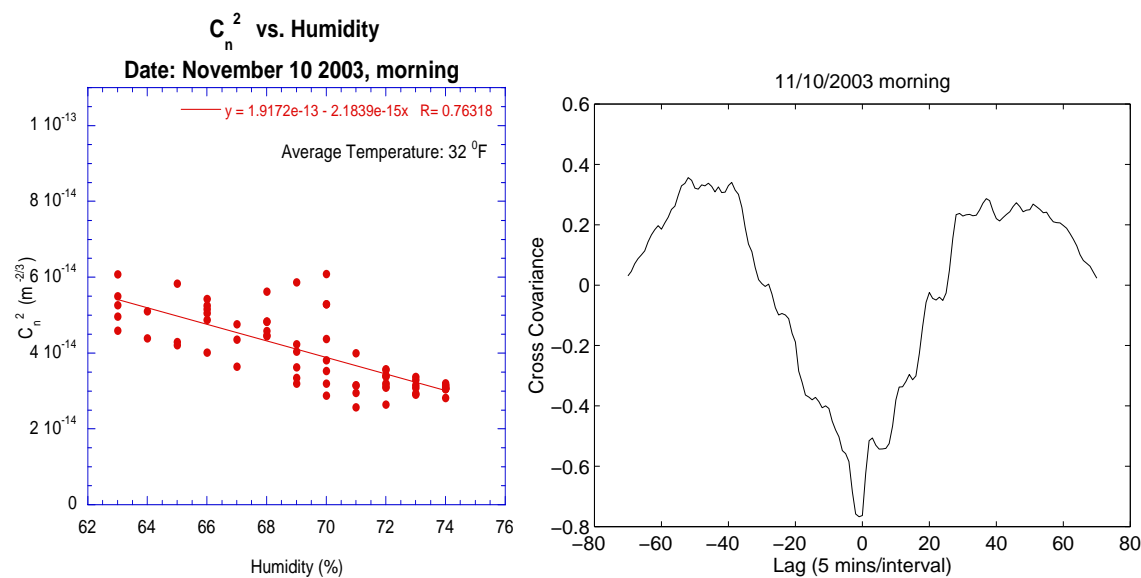

Figure 3. Nov 9 2003, Morning
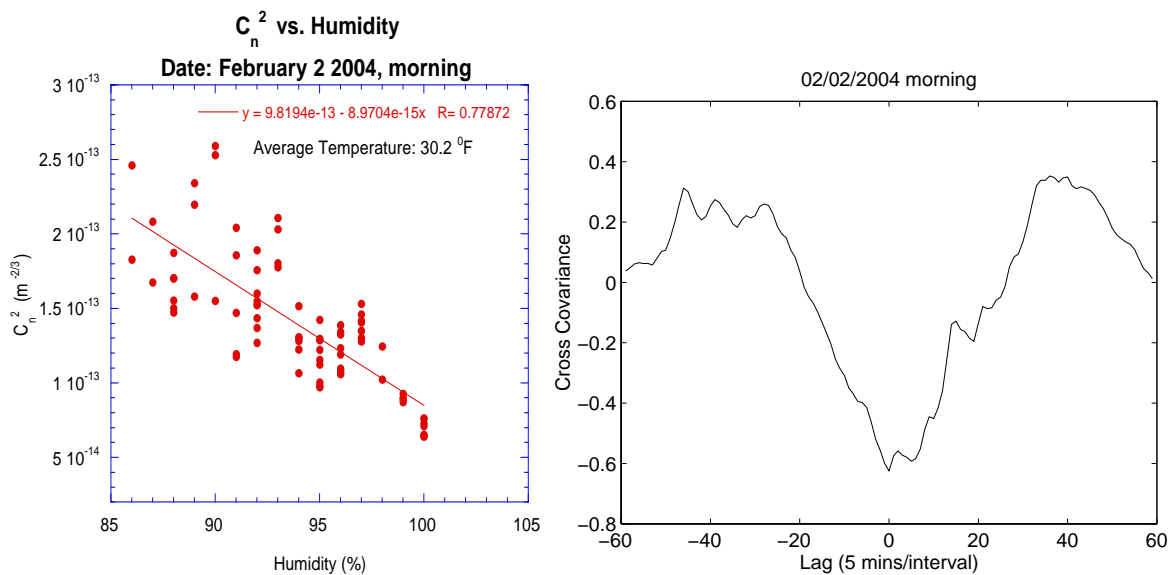
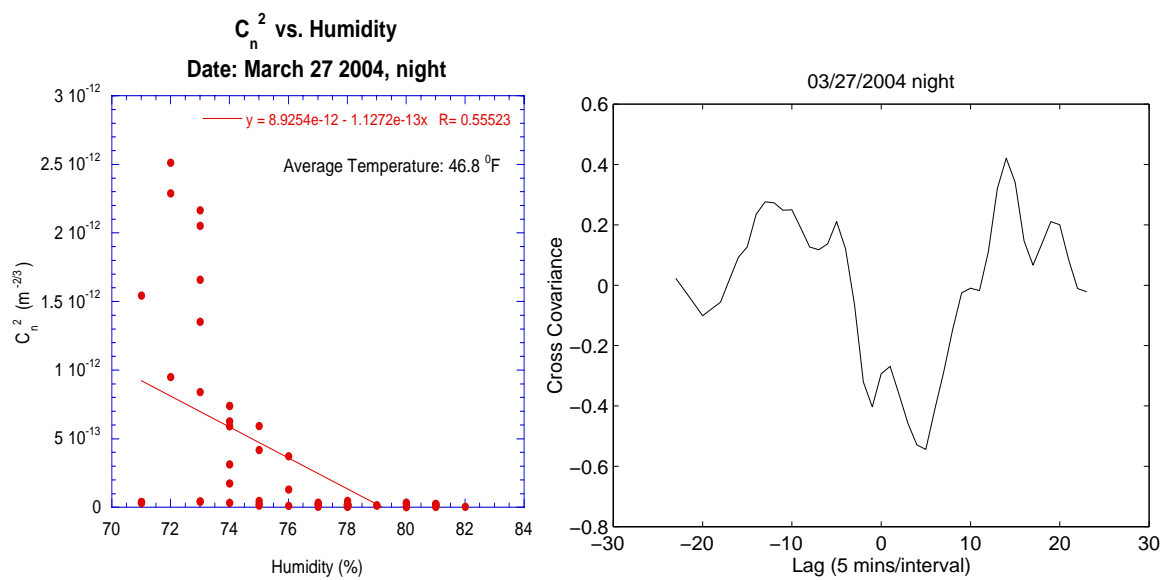**Figure 4.** Nov 10 2003, Morning

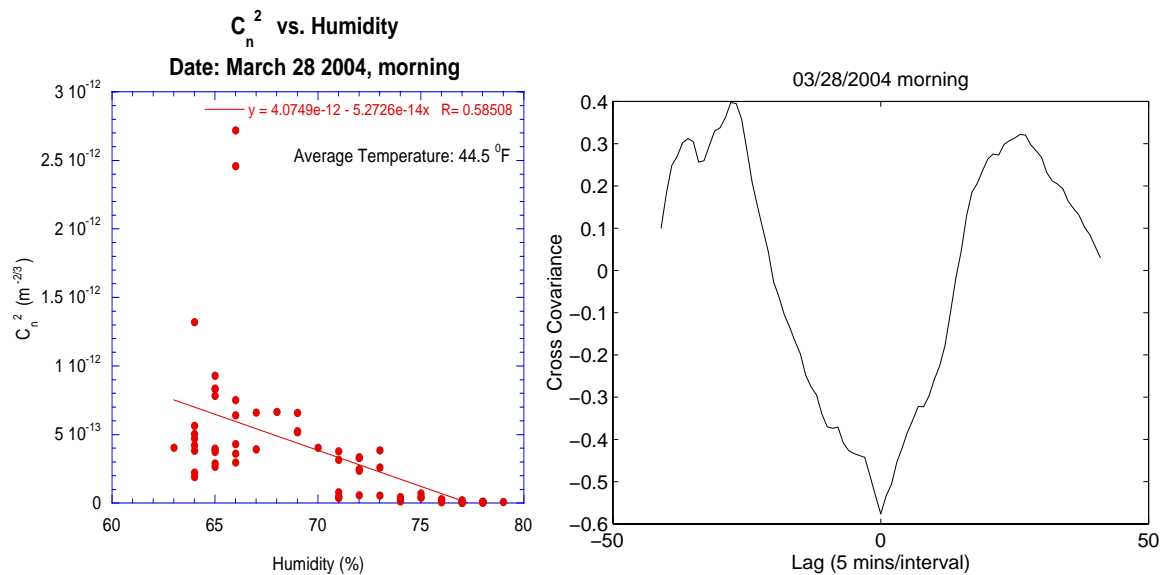**Figure 5.** Feb 2 2004, Morning

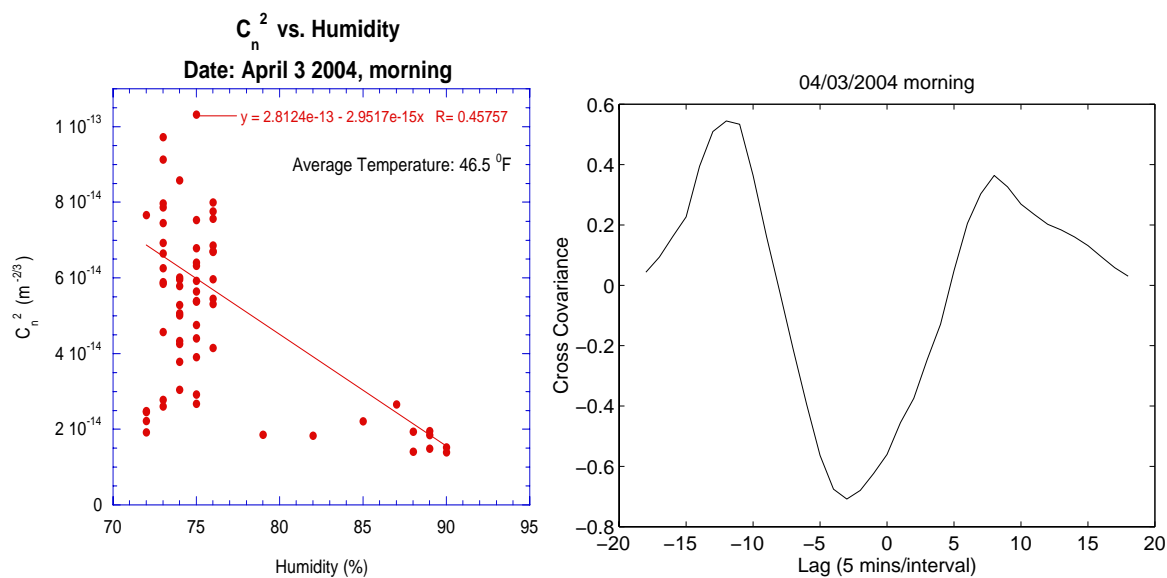**Figure 6.** Mar 27 2004, Night

Figure 7. Mar 28 2004, Morning

Figure 8. Apr 3 2004 Morning

Figure 9. Apr 3 2004, Night
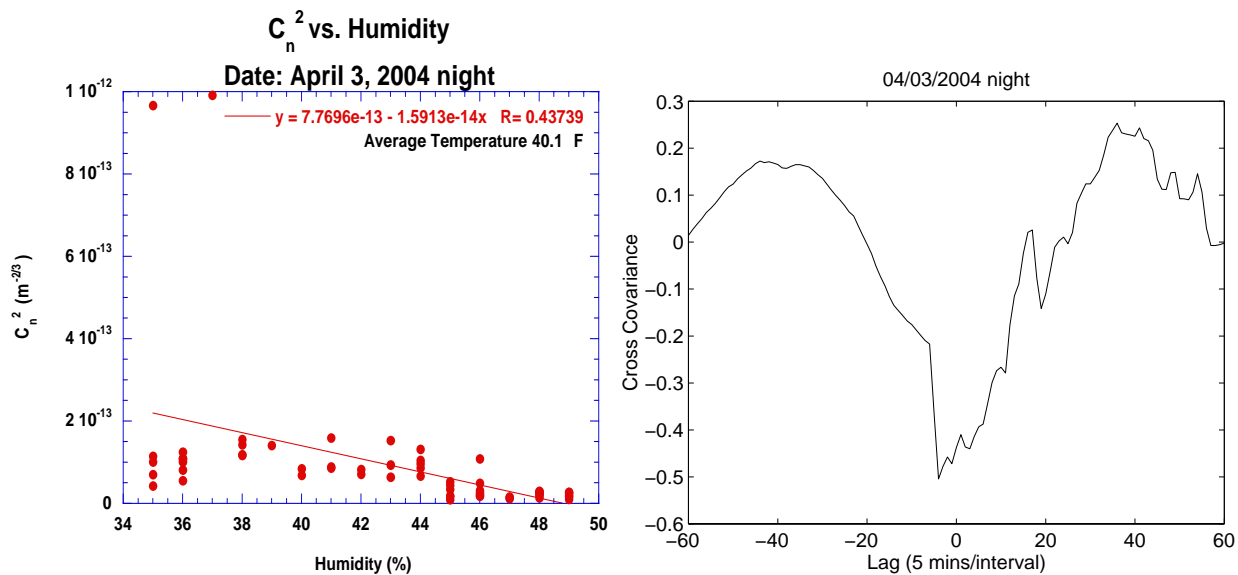
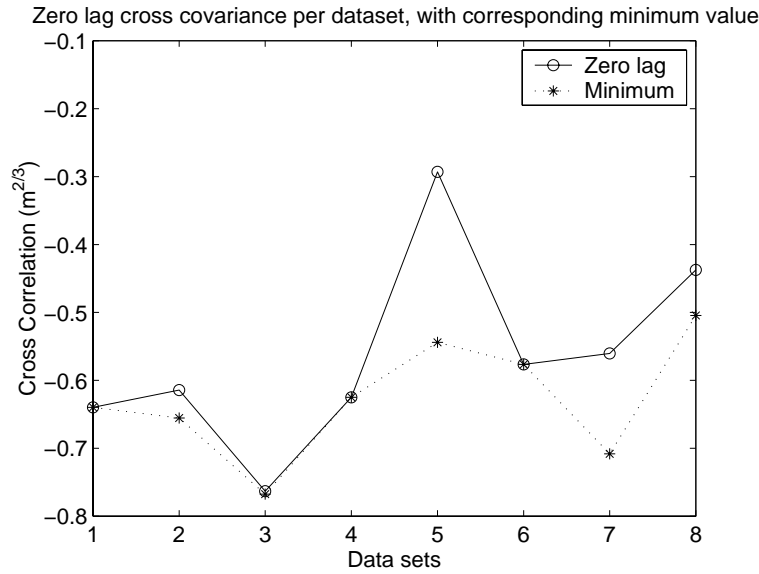Zero lag cross covariance per dataset, with corresponding minimum value

Figure 10. Cross covariance at zero lag. The data set numbers are defined in Table (1).

Fig.(5) The correlogram is evenly dispersed and the cross covariance is symmetric although spread.

Fig.(6) A greater dispersion is seen between 72% and 74% humidity than along the rest of the trendline, although in terms of magnitude it is not very large. The cross correlation is highly asymmetric with the minimum offset from the zero lag position.

Fig.(7) A greater dispersion is seen below 67% humidity than along the rest of the trendline (again the magnitude is not larger than the other plots).

Fig.(8) A large cluster of weakly correlated points are seen below 77% humidity. The cross covariance minimum is offset from the zero lag position but is otherwise reasonably symmetric.

Fig.(9) The correlogram shows a reasonably good correlation between the data series, although the cross covariance shows less symmetry than might be expected from the correlogram.

3.2. Covariance at zero lag, $C_{UV}(\delta t = 0)$

The C_{UV} at zero lag (i.e. when both C_n^2 and humidity data sets are totally overlapping) are given in Figure (10) and Table (1). The evidence for a negative correlation of humidity with C_n^2 is extremely strong; where the minimum cross covariances are different to the zero lag value, there is a time lag offset of no more than 25 minutes (equal to 5 sample points). Some of the offset error is possibly due to a timing mismatch between the clocks used for the DP+ and the LOA-004 instruments; even without accounting for this, the $C_{UV}(\delta t = 0)$ is still strongly negative.

The cross covariance method has provided unequivocal measures that the humidity and C_n^2 functions are negatively correlated.

4. CONCLUSIONS

Using empirical data, we have conclusively demonstrated that a strong negative correlation exists between the humidity and C_n^2 readings from experimental runs at the Naval Research Laboratory's Chesapeake Bay Detachment, for path lengths of about 100-m with relatively constant pressure, temperature and windspeed.

Data set number	Date [mm/dd/yyyy]	Zero timelag cross covariance
1	11/03/2003 night	-0.6397
2	11/09/2003 morning	-0.6144
3	11/10/2003 morning	-0.7632
4	02/02/2004 morning	-0.6251
5	03/27/2004 morning	-0.2930
6	03/28/2004 morning	-0.5764
7	04/03/2004 morning	-0.5604
8	04/03/2004 night	-0.4374

Table 1. The cross correlation datasets.

On the basis of this we suggest that C_n^2 is an inverse function of humidity in the absence of solar insolation at coastal sites.

We are currently in the process of taking equivalent data at UPR-Mayagüez and we expect to obtain measurements in the much less humid environment of New Mexico. With the availability of more data, we will be able to ascertain in a quantitative fashion the humidity contribution to C_n^2. We anticipate that a much deeper understanding of C_n^2 will be found from analysis of the complete data obtained under these extrememly varied ambient conditions.

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